

Course Syllabus

Douglas Marshall
January 5, 2022

PHILOSOPHY OF
MATHEMATICS

1 Logistics

TITLE: PHIL 236 - Proof, Knowledge, and Understanding in Mathematics

MEETING TIMES: Winter 2022, T TH 10:10–11:55 a.m.

MEETING LOCATION: Center for Math and Computing (CMC) Room 206

INSTRUCTOR: Douglas Marshall (dmarshall@carleton.edu)

INSTRUCTOR'S OFFICE: Leighton Hall 310

OFFICE HOURS: Tuesdays and Thursdays, 12:55–1:55 p.m. and by appointment

(Please book office hours using my Google appointment calendar. It's on the course website. In the beginning of the term, I will conduct office hours via Zoom.)

COURSE WEBSITE:

<https://moodle.carleton.edu/course/view.php?id=37858>

2 Texts

You must purchase or otherwise obtain reliable access to a copy of the following book:

Imre Lakatos (John Worrall & Elie Zahar, Eds.) *Proofs and Refutations* (Cambridge: Cambridge University Press, 1976)

Copies of *Proofs and Refutations* should be available at the Carleton College bookstore and should cost approximately \$25. Rental or other options may push the price as low as \$5. All other required readings will be available free of cost on the course website.

3 Course Requirements

1. Reading: It is crucial to the functioning of this course that you read the required texts carefully. If readings are assigned for a given class meeting, you should be sure to do the readings before class meets. Reading philosophical texts is hard. You will not be prepared if you don't do the reading carefully and critically. I strongly recommend doing every reading for the class at least twice.

2. Writing: There will be three kinds of writing assignment for this course. First, in most weeks after this week you will write **weekly responses** to questions about our readings. Your weekly responses will be 150–300 words in length and are to be posted to the course website by 3 p.m. on Wednesdays. They will form the basis of our discussions in the later part of Thursday's meeting. Second, there will be **two longer papers**, a midterm paper of approximately 5–6 pages and a final paper of approximately 8 pages. Third, there will be a **final paper proposal**, also of 150–300 words in length, in which you explain the question your final paper will address, the way you propose to answer that question, and the sources you plan to draw upon.

3. Class Participation: Participation includes punctual attendance in class and contributions to class discussions. I will take attendance at each meeting. If at all possible, you should avoid missing more than one week of class meetings. I encourage everyone to be part of our class discussions, and I suggest you set a goal of participating actively—like asking a question or describing your response to our readings—once per meeting (though it is fine if this often happens in a small group discussion).

It is possible that at some point you may need to attend the class via Zoom. In that case, I ask that if possible, you leave your camera on during the class meeting. I reserve the right to give less than full attendance credit to students on Zoom who appear to be away or unresponsive.

Note: There are no tests or exams; there is **no final exam**.

4 Grading

Weekly Responses (150–300 words): 20%

Midterm paper, 5–6 pages (roughly 1,500–1,800 words): 20%

Final Paper Proposal (150–300 words): 10%

Final Paper, 8 pages (roughly 2,400 words): 35%

Participation: 15%

I will use the percentages above in computing a final raw score at the end of the semester for each student. Your final grade will be based on your raw score, but it will take improvement over the course of the academic term into account.

Note: The deadlines for weekly responses are hard deadlines. If you hand in a weekly response after the deadline, you may not receive credit for it. However, once during the term you may skip a weekly response with no penalty to your grade in the course.

Extensions Policy: You may hand in the midterm paper and/or final paper for this class up to 24 hours late without penalty to your grade. When I calculate final grades, I will simply ignore up to 24 hours of lateness for the midterm paper, and up to 24 hours of lateness for the final paper.

Aside from the automatic 24 hour extension, you may request an ordinary extension of a paper deadline so long as you make your request *before* the day it is due. Work that is late without an extension or beyond an extension will be discounted at the rate of one-third of a grade per day (A to A-, A- to B+, and so forth). Papers will not be accepted more than one week after the due date except under extraordinary circumstances. In such circumstances, please get in contact with your class dean so they can help me to determine a reasonable paper extension.

5 Tentative Schedule of Topics and Assignments

The following weekly schedule is approximate and subject to change. Please see the course website for up-to-date weekly readings and assignments. After this week, the readings for a given week will always be posted by Friday evening of the previous week.

WEEK	TOPICS	AUTHORS
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1	Standard and Rationalist Conceptions of Mathematics	J. Brown Kant
2	Polya's Heuristics and Lakatos's Case Study	G. Polya I. Lakatos
3	Lakatos's Philosophy of Mathematics	I. Lakatos
4	Lakatos's Philosophy of Mathematics	I. Lakatos
5	Kant and Intuitionism	Kant L. E. J. Brouwer
6	Truth and Proof for Platonists	M. Resnik W. Tait
7	Revolutions in Mathematics	M. Crowe J. Dauben
8	Explanation in Mathematics	Aristotle P. Mancosu
9	Explanation, Generalizability and Unification	M. Steiner P. Kitcher
10	Benacerraf's Challenge	P. Benacerraf

WEEK	ASSIGNMENT	DUE DATE
1	—	—
2	Weekly Response	January 12
3	Weekly Response	January 19
4	Weekly Response	January 26
5	Midterm Paper	February 4
6	Weekly Response	February 9
7	Weekly Response	February 16
8	Final Paper Proposal	February 23
9	Weekly Response	March 2
10	Final Paper	March 14

6 Continuity of Instruction in a Time of Crisis

We are living in a difficult time. COVID-19 is interfering with our ability to be at college in the normal way. Deeply disturbing racial injustices are taking place, and so is social and political unrest. In times like these, we will need to exhibit flexibility with each other throughout the term.

I have done my best to design the course so that everyone can be successful, even if illness or external factors beyond our control mean that some of us may have to be absent for some of the class. If your personal situation is affecting your ability to engage with the course, please contact the Dean of Students Office so we can work towards reasonable accommodations. If technological problems are limiting your ability to participate, please contact the ITS Helpdesk at 507-222-5999 or helpdesk@carleton.edu.

If I come down with COVID-19 or another serious illness without severe symptoms, I may need to teach one or more of our classes remotely using Zoom. Similarly, if students come down with COVID-19 or other serious illnesses without severe symptoms, they may need either to attend remotely (via Zoom) or to obtain recordings of class meetings.

7 Privacy

Audio or video recordings of our class may from time to time be made either by me or by students for purposes of study or review. Class members should not share, replicate, or publish the recordings, in whole or in part, or use the recordings for any other purpose than for class-related studying. Recordings of class sessions that include student participants should not be saved or used past the end of the term. I will delete any audio or video recordings that include students within one month after the academic term is over. In return, I ask that you not share videos created for purposes of this course.

8 The Writing Center

The Writing Center provides a space staffed with peer writing consultants who can work with you during any stage of the writing process (brainstorming to final proofreading). Hours and more information can be found on the writing center website (<https://apps.carleton.edu/campus/asc/writingcenter/>). You can reserve specific times for conferences by using their online appointment system (<https://writingcenter.carleton.edu/>).

9 Accommodations for Students with Disabilities

Carleton College is committed to providing equitable access to learning opportunities for all students. The Office of Accessibility Resources (Henry House, 107 Union Street) is the campus office that collaborates with students who have disabilities to provide and/or arrange reasonable accommodations. If you have, or think you may have, a disability (e.g., mental health, attentional, learning, autism spectrum disorders, chronic health, traumatic brain injury and concussions, vision, hearing, mobility, or speech impairments), please contact OAR@carleton.edu or call Sam Thayer ('10), Director of the Office of Accessibility Resources (x4464), to arrange a confidential discussion regarding equitable access and reasonable accommodations.

10 Academic Integrity

All work submitted by you is assumed to be your own original work that has not been submitted elsewhere. Any words or ideas borrowed from other sources must be properly attributed. Any cases of suspected dishonesty will be forwarded to the Academic Standing Committee, as required by Carleton's policy on academic integrity. In confirmed cases of academic dishonesty, I will recommend a penalty ranging from a failing grade for the assignment to failure in the course. Carleton College may pursue further action.

For more information about academic integrity at Carleton and guidelines about how to avoid plagiarism in your work, please go to:
<https://www.carleton.edu/writing/plagiarism/>.

11 Course Description

PHIL 236 is a first course in the philosophy of mathematics. While there are no formal prerequisites, much of the course will involve reading, understanding, and reflecting philosophically on mathematical proofs.

We will focus primarily on the following topics in the philosophy and history of mathematics:

1. Proof and Knowledge: It seems obvious that proof plays a central role in generating mathematical knowledge. But what does a mathematical proof prove? Do we know that the theorem proved is true? Do we only know that the theorem would be true in any structure that renders the relevant

axioms true, perhaps without knowing whether there is such a structure? Are mathematical theorems beyond all legitimate dispute, or are they liable to be refuted by counterexamples, or rejected because of the kind of reasoning their proofs employ?

2. The Growth of Mathematical Knowledge: At first glance, mathematics appears to be a cumulative discipline piling theorem on top of theorem. On the other hand, when one reads historical mathematics, it often looks extremely different from present day mathematics and isn't rigorous by current standards. Is mathematics really cumulative? Is historical mathematics mathematics? How do we understand the growth of mathematical knowledge over time?

3. Explanatory Virtues of Proofs: Mathematicians do not in general stop with a single proof of a theorem but rather attempt to re-prove the same theorem over and over in different ways. This practice suggests that a primary goal of mathematics is not merely to *know* that some result holds, but rather to *understand* why it holds. But what is it for a proof to explain, or provide understanding, of the theorem that it proves? What (if any) are the distinguishing features of proofs that explain their results as opposed to merely establishing them?

12 Course Objectives

A. Learn how to structure and write a paper.

B. Learn how to articulate and defend a philosophical position concerning mathematics in speech and in writing.

C. Know how to research or continue pursuing topics in the history and philosophy of mathematics after the class is over (if you wish).