Proof, Knowledge, and Understanding in Mathematics

⚙ ▾

Turn editing on

Instructor: Douglas Marshall (dmarshall@carleton.edu)

Meeting Times: Winter 2022, Tuesdays and Thursdays, 10:10–11:55 a.m.

Meeting Location: Center for Math and Computing (CMC) Room 206

Office Hours: Tuesdays and Thursdays, 12:55–1:55 p.m., and by appointment. (Wednesday afternoons are likely to be good for appointments.) Please book office hours meetings in My Google Calendar Appointment Page. Until further notice, Office Hours take place in my Office Hours Zoom Room (passcode: 766127).

🗨 Announcements

📕 Course Syllabus Uploaded 6/01/22, 08:38

📕 Course Microsyllabus Uploaded 23/12/21, 12:29

Hidden from students

🌐 My Google Calendar Appointment Page

🌐 Office Hours Zoom Room

Submit Your Papers Here

📄 Midterm Paper   ☑

📄 Final Paper   ☑

Resources for Writing Your Papers

📄 Jim Pryor's Guidelines on Writing a Philosophy Paper

📕 Some Topics for the Midterm Paper Uploaded 1/05/19, 09:59

📕 Sample Midterm Paper (please don't distribute) Uploaded 27/01/17, 09:24

📕 Weekly Response Grading Rubric Uploaded 16/01/22, 13:23

Launchpad for Research and Further Reading

📕 Ways to Go On in the Philosophy of Mathematics Uploaded 1/06/19, 16:12

🌐 Foundations Interest Group at the U of M

🌐 Carleton College Philosophy Department

🌐 Stanford Encyclopedia of Philosophy

🌐 Wi-Phi Philosophy Videos

🌐 PhilPapers

You can use search terms or browse by topic (and philosophy of mathematics is one of the topics).

⬆

🌐 Philosopher's Index (Carleton Access)

   Search for philosophy books and articles.

[Week 1] Standard and Rationalist Conceptions of Mathematics

Welcome!

Readings and Activities for Thursday

1. Read J. Brown _Philosophy of Mathematics_ "Introduction" (8 pages)

Questions for the reader: What, according to Brown, is the standard image of mathematics? What are the aspects Brown lists, and what does he mean by them? Which aspects of the standard image fit with your understanding of mathematics, and which don't? Which aspects of the standard image do you think are part of your own mathematics education, and which aren't?

2. Read Kant _Critique of Pure Reason_ Introduction (B), pp. 136-148 (13 pages). Focus especially on Section I through Section III, pp. 136-141. If you find the remaining sections difficult, you may skim them and just focus on the places where Kant is discussing mathematics.

Questions for the reader: What does Kant mean by "cognition a priori"? What exactly is Kant's distinction beween cognition that is a priori and cognition that is empirical? Do you agree with Kant that mathematical cognition is a priori? Does Kant provide you with any argument to convince you of his view about mathematical cognition?

There is no weekly response due this week. We'll start those next week. So far, we're just reading, thinking, and getting to know something about the course and each other.

📕 J. Brown _Philosophy of Mathematics_ "Introduction"

📕 Kant _Critique of Pure Reason_ Introduction (B) Uploaded 2/01/17, 23:54

📕 Handout on the Standard Image and Rationalist Conception of Math Uploaded 8/04/19, 14:35

📕 PHIL 236 Interest Sheet Uploaded 6/01/22, 09:33

Dive Deeper / Optional Extras

📕 Six Proofs of the Infinitude of Primes from _Proofs from the Book_

[Week 2] Polya and Lakatos on Mathematical Heuristic

Readings and Activities for Tuesday:

1. Read Polya "Induction in Solid Geometry", p. 35-43 (optional: pp. 52-58).

Questions for the Reader: What do you make of Polya's advice about how to determine a relationship between the vertices, edges, and faces of polyhedra? Why is Polya describing the way mathematicians investigate a question in a way that is very similar to the way (he thinks) empirical scientists investigate a question? How is Polya suggesting we weigh the evidence in favor of the conjecture that Vertices - Edges + Faces = 2 at various stages of the investigation?

2. Read Lakatos _Proofs and Refutations_, pp. 1 - 15 (read to the end of Chapter 1 Section 4 a). If you don't have the book yet and need a PDF version you can read, please click on the Course Bookshelf below to get the PDFs.

Readings and Activities for Thursday:

1. Read Lakatos _Proofs and Refutations_, pp. 15-35 (read to the end of Chapter 1 Section 4 Subsection d)

Please post your weekly response using the moodle forum below. Your response is due by 3 p.m. on Wednesday and should be between 150 and 300 words.

📒 First Weekly Response Forum

📕 Polya "Induction In Solid Geometry"

📕 Polya Handout Uploaded 11/01/22, 08:54

📕 Lakatos Handout Uploaded 11/01/22, 08:54

Dive Deeper / Optional Extras

📕 Abigail Kirk "Euler's Polyhedron Formula" (Cauchy Proof) Uploaded 25/01/17, 10:41

📕 Forum Thread: Lakatos's Teacher on Proof Uploaded 13/01/22, 15:49

📕 Forum Thread: Definition P Uploaded 13/01/22, 15:49

📕 Forum Thread: Confirmation By Instances in Mathematics Uploaded 13/01/22, 15:49

[Week 3] Lakatos's Proofs and Refutations I

Readings for Tuesday: Lakatos _Proofs and Refutations_, up through the end of Chapter 1, Section 5 ("Criticisms of the Proof-Analysis...")

Readings for Thursday: Lakatos _Proofs and Refutations_, up through the end of Chapter 1, Section 6 ("Return to Criticism...")

📒 Second Weekly Response Forum

📕 Some Methodologies Described in _Proofs and Refutations_ Uploaded 16/01/17, 15:47

Dive Deeper // Optional Extras

📕 Forum Thread: Vicious Infinity Uploaded 21/01/22, 17:17

📕 Forum Thread: Discovery and Justification in Mathematics Uploaded 21/01/22, 17:17

📕 Forum Thread: Language and Proof Uploaded 21/01/22, 17:18

[Week 4] Lakatos's Proofs and Refutations II

Readings for Tuesday: Lakatos _Proofs and Refutations_, until the end of Chapter 1, Section 8.

Readings for Thursday: Lakatos _Proofs and Refutations_, until the end of Section 9. In other words, finish Chapter 1.

📒 Third Weekly Response Forum

Dive Deeper // Optional Extras

📕 Forum Thread: Certainty and Meaning Uploaded 28/01/22, 21:23

📕 Forum Thread: Mathematical Depth Uploaded 28/01/22, 21:23

📕 Forum Thread: Different Proofs of the Same Theorem Uploaded 28/01/22, 21:24

[Week 5] Kant, Brouwer, and Constructivism

Readings and Activities for Tuesday

1. Read [Immanuel Kant "The Discipline of Pure Reason"](#) (14 pages)

2. Have a look at Euclid's Elements online here (diagrams may only work in certain browsers such as Chrome):

http://aleph0.clarku.edu/~djoyce/java/elements/

Kant keeps talking about the angle-sum theorem for triangles, which is Euclid I.32. You can find the theorem and proof here so you know what Kant has in mind:

http://aleph0.clarku.edu/~djoyce/java/elements/bookI/propI32.html

Readings and Activities for Thursday

1. Read [James Robert Brown "Constructive Approaches"](#) (18 pages)

2. Bring whatever ideas, outline, or draft you have of your [midterm paper](#) to class. I'm planning to devote time in the later part of class for you to discuss your paper ideas with each other in small groups.

As a reminder, there's no weekly response for this week, but the [midterm paper](#) is due February 4.

[Immanuel Kant "The Discipline of Pure Reason"](#)

[Handout on Kant's Philosophy of Mathematics](#) Uploaded 1/05/19, 10:00

[James Robert Brown "Constructive Approaches"](#) Uploaded 28/01/22, 18:20

Dive Deeper // Optional Extras

[L.E.J. Brouwer Selections](#)

[Week 6] The Truth-Proof Problem for Mathematical Realism

Reading for Tuesday and Thursday: [Michael Resnik "Proof as a Source of Truth"](#)

I have tried to keep the reading load lighter this week, but I wanted to post an optional reading you may be interested in that is relevant to the article by Resnik. If you get interested in the truth-proof problem for platonism, I suggest reading [William Tait "Truth and Proof: The Platonism of Mathematics"](#).

Note that there will be a Weekly Response due on Wednesday of this week.

[Michael Resnik "Proof as a Source of Truth"](#)

[Fourth Weekly Response Forum](#)

[Handout on Metaphysics of Mathematics](#) Uploaded 11/02/22, 11:45

Dive Deeper // Optional Extras

[William Tait "Truth and Proof: The Platonism of Mathematics"](#)

[Forum Thread: Projecting Properties](#) Uploaded 22/02/22, 08:57

[Forum Thread: Resnik's Realism](#) Uploaded 22/02/22, 08:58

[Forum Thread: Structuralism About Everything](#) Uploaded 22/02/22, 08:58

[Week 7] Revolutions and the Growth of Mathematical Knowledge

Readings for Tuesday: Thomas Kuhn "The Nature and Necessity of Scientific Revolutions" ; Michael Crowe "Ten 'Laws'"

Readings for Thursday: Joseph Dauben "Conceptual Revolutions and the History of Mathematics" (pp. 49-71 in the PDF)

Note: I have included a follow up piece by Dauben (Appendix (1992)) in the PDF of Dauben's paper. It is strictly optional. I'm including it in case the additional case studies Dauben talks about are of interest to you.

📕 Thomas Kuhn "The Nature and Necessity of Scientific Revolutions" Uploaded 10/05/19, 10:15

📕 Michael Crowe "Ten 'Laws'" Uploaded 10/02/17, 15:40

📕 Joseph Dauben "Conceptual Revolutions and the History of Mathematics" Uploaded 10/02/17, 15:40

💬 Fifth Weekly Response Forum   ☑

Dive Deeper // Optional Extras

I put together an Outline of Kuhn's Structure of Scientific Revolutions you may find helpful to get an overview of Kuhn's view.

📕 Outline of Kuhn's Structure of Scientific Revolutions (2019) Uploaded 15/02/22, 08:39

📕 Forum Thread: Daubens on Cumulativity Uploaded 22/02/22, 08:55

📕 Forum Thread: Rigor and Historical Mathematics Uploaded 22/02/22, 08:55

📕 Forum Thread: Revolutions in Mathematics Uploaded 22/02/22, 08:55

[Week 8] Mathematical Explanation I

Your Final Paper Proposals are due by 3 p.m. on Wednesday so we can devote the latter part of Thursday's meeting to a workshop on your final papers.

Reading for Tuesday:

Paolo Mancosu "Explanation in Mathematics" (from the Stanford Encyclopedia of Philosophy), Section 2 and Sections 4 - 7 (pp. 5-9, 30-45)

Reading for Thursday:

Aristotle Posterior Analytics A1 - A13,

Aristotle Physics Book II Selections (PDFs below).

The Aristotle readings are difficult, and I recommend you get a head start on them if you can. We will be focussing on some of the things Mancosu discusses in his SEP article, so it would be good to keep those issues in mind as you are reading Aristotle.

📕 Paolo Mancosu "Explanation in Mathematics" Uploaded 17/05/19, 21:36

📕 Aristotle Posterior Analytics A1 - A13 Uploaded 17/02/17, 20:54

📕 Aristotle Physics Book II Selections Uploaded 17/02/17, 20:54

📕 Aristotle Handout Uploaded 24/02/22, 11:35

💬 Final Paper Proposal Forum   ☑

Dive Deeper // Optional Extras

One issue right next door to mathematical explanation concerns the question of why mathematicians re-prove theorems over and over. If that issue sounds interesting to you, I recommend the paper below. I have also added a brief handout going over Dawson's answers.

📕 John Dawson "Why Do Mathematicians Re-prove Theorems?" Uploaded 23/02/17, 15:31

📕 Dawson Re-Proving Theorems Handout Uploaded 24/05/19, 10:21

[Week 9] Mathematical Explanation II

Reading for Tuesday: Mark Steiner "Mathematical Explanation"

Reading for Thursday: Philip Kitcher "Explanatory Unification"

Note: These articles contain a mix of examples, some very elementary (say, from arithmetic and geometry), some very advanced. You should feel free to focus on the philosophical accounts of mathematical explanation and the more elementary examples given in support or in refutation of them, skipping or skimming the more advanced examples.

📕 Mark Steiner "Mathematical Explanation"

📕 Philip Kitcher "Explanatory Unification"

💬 Sixth Weekly Response Forum   ☑

📷 Class Blackboard of Explanatory and Non-explanatory Proofs Uploaded 27/05/19, 16:30
    [Hidden from students]

📕 PHIL236 Explanatory Proofs (From the Blackboard in 2022) Uploaded 7/03/22, 14:10

📕 PHIL236 Explanatory Proofs (From the Blackboard in 2019) Uploaded 7/03/22, 14:12

Dive Deeper // Optional Extras

📕 Forum Thread: Aristotle's Four Causes Uploaded 7/03/22, 14:07

📕 Forum Thread: Current Best Account of Explanation Uploaded 7/03/22, 14:07

📕 Forum Thread: Steiner Vs. Kitcher Uploaded 7/03/22, 14:08

[Week 10] Benacerraf's Challenge and What is Philosophy?

Reading for Tuesday:

1. Read Paul Benacerraf "Mathematical Truth".

Readings and Activities for Thursday:

1. Read Bertrand Russell "The Value of Philosophy". Or, if you prefer, you can listen to it read aloud (see below for a link to the audio file).

2. Read Wilfred Sellars "The Philosophical Quest". Or, if you prefer, you can listen to it read aloud (see below for a link to the audio file).

3. If you think you'd like to go deeper into the philosophy of mathematics, have a look at the document Ways to Go On in the Philosophy of Mathematics.

4. If you think you'd like to go broader and pursue other parts of philosophy, have a look at the current list of courses in the Carleton philosophy department.

📕 Paul Benacerraf "Mathematical Truth" Uploaded 3/05/19, 16:50   ⬆

Bertrand Russell "The Value of Philosophy" Uploaded 9/02/22, 12:17

Russell's "Value of Philosophy" Read Aloud

Wilfred Sellars "The Philosophical Quest" Uploaded 9/02/22, 12:17

Sellars's "Philosophical Quest" Read Aloud

Current List of Courses in the Carleton philosophy department

End-Of-Term Writing Exercise Uploaded 8/03/22, 09:18

End-Of-Term Evaluation Form Uploaded 8/03/22, 09:14

If you weren't able to fill one out in class in person, it would be great if you could fill one out and return it to Kristen Askeland in the Religion Department Office. You could email it to her, and she would anonymize the result, or you could hand deliver it. Thanks!

Dive Deeper // Optional Extras

Sellars's "The Philosophical Quest" is the first part of a longer paper. If you find "The Philosophical Quest" intriguing, you could consider reading the longer paper below.

Wilfred Sellars "Philosophy and the Scientific Image of Man" Uploaded 31/05/19, 12:20

Course Bookshelf

Lakatos's essay _Proofs and Refutations_ was originally published in four installments in the _British Journal for the Philosophy of Science_. PDFs of the original publication are below. If you don't have a copy of the book yet, you may read from these PDFs. Just read up to the section number indicated in the assignment (and ignore the part about "Chapter 1").

LakatosProofsAndRefutationsBJPS I Uploaded 23/12/21, 17:49

LakatosProofsAndRefutationsBJPS II Uploaded 23/12/21, 17:49

LakatosProofsAndRefutationsBJPS III Uploaded 23/12/21, 17:49

LakatosProofsAndRefutationsBJPS IV Uploaded 23/12/21, 17:49

The book _Euler's Gem_ is an excellent historical companion to Lakatos's _Proofs and Refutations_. Here's a brief sample of it discussing Euler's proof.

Euler's Gem Account of Euler's Proof Uploaded 24/04/19, 15:27

The following files all describe historical or contemporary proofs of Euler's polyhedron formula. I recommend the article by Abigail Kirk for being especially easy to understand.

Abigail Kirk "Euler's Polyhedron Formula" (Cauchy Proof) Uploaded 25/01/17, 10:41

EulerDemonstratioNonnullarum1752bLatina

EulerDemonstratioNonnullarum1752bEnglish

CauchyRecherchesSurLesPolyedres1813French

CauchyRecherchesSurLesPolyedres1813English

Twenty Proofs of Euler's Polyhedron Formula Uploaded 1/05/19, 10:13

The following files are two of the very best critical and interpretive responses to _Proofs and Refutations_.

David Corfield "Lakatos's Philosophy of Mathematics" Uploaded 26/05/19, 08:31

Solomon Feferman "The Logical Structure of Mathematics" Uploaded 26/05/19, 08:33

🛈 Moodle Docs for this page

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Home

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  Gould Library
  Gould Guides
  BN Adoption and Insights
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  Carleton Moodle Docs
  Student Gateway